Inequality with areas of bisectorial triangles.
https://www.linkedin.com/feed/update/urn:li:activity:6749388990337609728
Let ABC be a triangle with side-lengths a, b, c inscribed in a circle with radius R and let I be it's incenter. Let F_a, F_b and F_c be the areas of the triangles BCI, CAI and ABI, respectively.
Prove that $\frac{1}{F_a^2} + \frac{1}{F_b^2} + \frac{1}{F_c^2} \geq \frac{16}{R^4}$.

Solution by Arkady Alt, **San Jose**, **California**, **USA**.
Let r be inradius of $\triangle ABC$. Since $F_a = \frac{ra}{2}$ then $\sum \frac{1}{F_a^2} = \sum \frac{4}{r^2 a^2} = \frac{4}{r^2} \sum \frac{1}{a^2}$ and, therefore, $\sum \frac{1}{F_a^2} \geq \frac{16}{R^4} \Leftrightarrow R^2 \sum \frac{1}{a^2} \geq \frac{4r^2}{R^2}$.
Since $a^2 + b^2 + c^2 \leq 9R^2$ and by Cauchy Inequality $\sum \frac{1}{a^2} \geq \frac{9}{a^2 + b^2 + c^2}$ then $R^2 \sum \frac{1}{a^2} \geq \frac{9R^2}{a^2 + b^2 + c^2} \geq 1$. Also we have $\frac{4r^2}{R^2} \leq 1$ due Euler's Inequality $R \geq 2r$.